PART I: Solve the following problems before you have fun with the simulation

1. A solid disk and a ring have the same mass and radius. Which will have the greater rotational inertia about an axis through its center? Explain.

A ring has a larger moment of inertia because its entire mass is concentrated at the rim at a maximum distance from the axis.

Moment of inertia

Moment of inertia is defined as the quantity expressed by the body resisting angular acceleration which is the sum of the product of the mass of every particle with its square of a distance from the axis of rotation.

It can be described as a quantity that decides the amount of torque needed for a specific angular acceleration in a rotational axis.

Moment of Inertia is also known as the angular mass or rotational inertia.

The SI unit of moment of inertia is kg m2.

It is important to understand that the moment of inertia depends on three factors, and they are:

Mass of the body

Shape, size, and density of the body

The position of the axis of rotation

2. Suppose a merry-go-round is rotating at the rate of 12 rev/min.

a. Express this rotational velocity in rev/s. 12/60

12/60

b. Express this rotational velocity in rad/s.

12x2x3.14/60

3. The rate of rotation for popular music records on a record player was 50 RPM.
   a. Express this rotational velocity in rev/s.

   b. Through how many revolutions does the record turn in a time of 5 s?

4. A wheel is spinning at 24 rad/s when a brake is applied, providing a constant angular acceleration of 3 rad/s^2.

a. How long does it take for the wheel to stop spinning?

b. Through what angle (θ) does the wheel rotate during this time?

5. A student, sitting on a stool rotating at a rate of 12 RPM, holds masses in each hand. When his arms are extended, the total rotational inertia of the system is 3 kg·m^2. He pulls his arms in close to his body, reducing the total rotational inertia to 1.2 kg·m^2. If there are no external torques, what is the new rotational velocity of the system?

6. A CD of radius 0.08 m speeds up its rotation uniformly from zero to 12 rad/s in 3 seconds.

a. What is its angular acceleration?

b. What is its angular velocity 5 seconds after it starts spinning?

c. What is the linear speed of a point on the edge of the CD 5 seconds after it starts spinning?

d. Find the ***centripetal*** acceleration of a point on the edge of the CD 5 seconds after it starts spinning.

e. Find the ***tangential*** components of the acceleration of a point on the edge of the CD 5 seconds after it starts spinning.

7. A bowling ball of mass 8 kg and radius 0.11 m rolls down the alley with an angular velocity of 80 rad/s. Assume it is a solid sphere ($I = mr^2$)
    a. What is its moment of inertia ***I***?

b. What is its angular momentum ***L***?

c. What is its rotational kinetic energy?

d. What is its translational kinetic energy?

e. What is its total kinetic energy?

8. A merry-go-round in the park has a radius of 2.5 m and a rotational inertia of 900 kg·m^2. A child pushes the merry- go-round with a constant force of 150 N applied at the edge and parallel to the edge.
   a. What is the net torque acting on the merry-go-round about its axle?

   b. What is the rotational acceleration of the merry-go-round?

   c. At this rate, what will the rotational velocity of the merry-go-round be after 15 s if it starts from rest?

9. A mass of 1 kg is located at the end of a ***pendulum*** 0.5 m in length. The pendulum is rotating about an axis at its opposite end with a rotational velocity of 12 rad/s.

a. What is the rotational inertia of the system?

b. What is the angular momentum of the system?

c. What is its kinetic energy?

10. A 2-kg bicycle tire with a radius 0.33 m rotates with angular speed of 120 RPM.
    a. Find the angular velocity in rad/s.

    b. Find the rotational kinetic energy.

    c. Find the angular momentum of the tire, assuming it can be modeled as a ring.

11. A disk-shaped merry-go-round with mass 80 kg and radius 1.2 m. There are 3 students each with mass 90 kg and standing at the rim (r=1.2 m).
    a. What is the rotational inertia of the merry-go-round? (use $I_{disk} = mr^2$)

    b. What is the rotational inertia of each student?

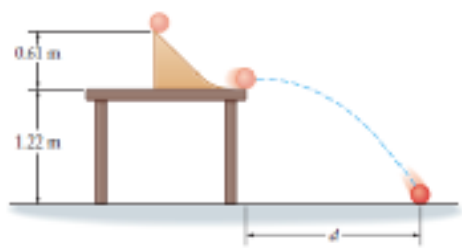
    c. What is total rotational inertia of the merry-go-round and 3 students?

    d. If two students jumped into the center (r=0.1m) and one student stays at r =1.2 m. What its total inertia?

    e. Before 2 kids jumped into the center, the *initial* angular velocity (ω_i = 10 rad/s). What is its *final* angular velocity (ω_f) after 2 kids jumped into the center and one kid stay at the rim r=1.2 m?

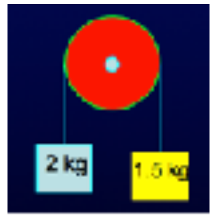
12. A massless rope is wrapped around the outside of a **disk** ($I = 0.5\ MR^2$) of mass 4 kg and radius 0.6 m. The sphere rotates about an axis passing through its center. If the tension in the rope is a constant 12 N, what is the angular acceleration of the sphere?

13. A massless string is wrapped 10 times around a disk of mass M = 5 kg and radius R = 0.12 m. The disk is constrained to rotate without friction about a fixed axis though its center. The string is pulled with a force F = 5 N until it has unwound. (Assume the string does not slip, and that the disk is initially not spinning).
    a. What is moment inertia of the disk?
    b. What is the torque the disk (τ)?
    c. What is angular acceleration (α)?
    d. What is angular velocity (ω) after the string has unwound?



14. A solid sphere is rolling down a ramp. Its radius is 0.02 m, and it has mass of 0.5 kg. It starts rolling from h = 0.61 m (see the figure above). Then it continues its track after the ramp. The ramp is 1.22 above the floor.
    a. What is its rotational inertia?
    b. What is its linear velocity when it leaves the ramp?
    c. How long does it take to hit the floor?
    d. Calculate the horizontal distance (d)?

15. A 2 kg mass and a 1.5 kg mass are attached to opposite ends of a string, which passes over a pulley with a mass of 200 kg and a radius of 0.2 m. The string does not slip on the surface of the pulley. Note: the pulley looks like a disk.



a. Find the linear acceleration of the pulley.

b. Find the angular acceleration of the pulley.

Torque, Moment of Inertia, and Angular Momentum
https://phet.colorado.edu/en/simulations/torque

Part II: Torque

1. Click the tab at the top that says torque
2. Set the force equal to 1 N.
3. Click Go let this run for at least 10 seconds
4. What is the torque on the wheel (include direction).
5. What eventually happens to the lady bug?

6. From Newton's second law, a force will cause an

7. When considering angular motion, a torque will cause an (consider both torque equations):

8. What must be the centripetal force that keeps the ladybug moving in a circle?

9. Why does this force eventually fail?

10. Reset all, and set the force back to 1 N.
11. Observe the acceleration vector as you start. Describe how it changes.

12. Will the acceleration vector ever point directly to the center? Explain why / why not. (the next steps might help you answer this question)

13. Reset all. Set the force back to 1 N.
14. Hit start, wait about 2 seconds, and set the brake force to 1 N. Hit enter and observe.
15. Describe the motion of the wheel.

16. What happened to the acceleration vector? Why?

17. What is the net torque?

18. Reset all. Set the Force back to 1 N. Hit Start.
19. After a few seconds, set the brake force equal to 3N and hit enter.
20. Right after you set the break force, calculate the net torque (check with the graph):
21. Eventually the disc stops and the net torque is zero. This is because the breaking torque changed as you can see in the graph. Why did it change?

Part III: Moment of Inertia

1. Click the Moment of Inertia Tab at the top.
2. Disregard any millimeter units. They should all be meters.
3. To best see the graphs, set the scale of the torque graph to show a range of 20 to -20.
4. Set the Moment of Inertia Graph to show a range of 2 kg m 2 to – 2 kg m 2
5. Set the angular acceleration graph to show 1,000 degrees / s 2 to –1000 degrees / s 2
6. Calculate the moment of Inertia for the disk with the given information.

7. Hold the mouse over the disk so the mouse finger is pointing anywhere between the green and pink circles.
8. Hold down the left mouse button. Move your mouse to apply a force.
9. Look at the graph and try to apply a force that creates a torque of 10.
10. Use the ruler to determine the radius at any point between the green and pink circles.

    r = ___________m

11. Calculate what the applied force must have been.

12. Calculate the angular acceleration of the disk. Work in SI units, and then convert to degrees / s^2. Compare to the graph to check your answer.

13. Predict what will happen to the moment of inertia if you keep the mass of the platform the same, but you create a hole in the middle (increase inner radius).

14. Set the inner radius equal to 2. Calculate the moment of inertia for this shape. Set the disk in motion and check your answer by looking at the moment of inertia graph.

15. Even when the force on the platform changes, the moment of inertia graph remains constant. Why?

16. Finish the following statements:

    When the mass of an object increases, the moment of inertia

    When the distance of the mass from the axis of rotation increases, the moment of inertia

Part IV Angular Momentum

1. Click the Angular Momentum tab at the top.
2. Set the scale of the moment of inertia and angular momentum graphs to show a range of 2 to -2.
3. Set the angular speed to be 45 degrees / s.
4. What is the SI unit for angular momentum?

5. Calculate the angular momentum in SI units (you should have already calculated the moment of inertia in part II).

6. While the disk is moving, change the inner radius to 2.
7. Observe the graphs.
8. Changing the inner radius automatically changes the angular velocity to 36 degrees / s. Why? (mention moment of inertia and angular momentum in your answer).